Remarks on IEEE 802.11 DCF Performance Analysis

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***Abstract*— This letter presents a new approach to evaluate the throughput/delay performance of the 802.11 Distributed Coordination Function (DCF). Our approach relies on elementary conditional probability arguments rather than bidimensional Markov chains (as proposed in previous models), and can be easily extended to account for backoff operation more general than DCF's one.**

***Index Terms*— IEEE 802.11, Multiple Access Control, performance evaluation.**

I. Introduction

A SIMPLE, but accurate, analytical model to evaluate the IEEE 802.11 Distributed Coordination Function (DCF) saturation throughput performance was proposed in [1] and further detailed in [2]. Several papers have built on this basic model, e.g. adapting it to backoff variants [3], [4], using different assumptions [5], or accounting for supplementary modeling details such as finite retransmission attempts [6], k-ary exponential backoff and multiple traffic classes [7], error-prone channel conditions [8], hidden terminals [9], etc. In addition to the throughput analysis, some of the above mentioned papers provide a companion derivation of the average delay performance. Indeed, a careful derivation is needed in the case of finite retransmission attempts [10].

The contribution of this letter is threefold. Section III presents an alternative and simpler derivation of the model proposed in [1], based on elementary conditional probability arguments rather than bidimensional Markov chains. The new derivation clearly decouples the backoff stage updating process from the backoff counter one. It also highlights the ability of the analysis to account for a large variety of backoff mechanisms, differing in terms of backoff stage evolution and per-stage backoff counter extraction rules. Section IV modifies the throughput computation in [1] to properly account for the backoff freezing details specified in the 802.11 DCF standard. Finally, section V proposes an alternative derivation of the average delay performance, based on a perhaps not obvious application of the Little's result.

II. Assumptions and Notations

In what follows, we assume the reader to be familiar with the analysis proposed in [1], [2]. This analysis considers ideal channel conditions, and assumes a finite and fixed number n of contending stations in "saturation" conditions, i.e. each station is assumed to always have packets in the transmission buffer. Such a model was enabled by the recognition that each individual station sees the events occurring on the channel according to a discrete, though not uniform, slotted time scale. With respect to [1], we here adopt a more precise definition of the slotted time scale. Specifically, a new "model" slot starts whenever the backoff counter of a non transmitting station is decremented. As discussed in more details in section IV, a single model slot may comprise more than a single transmission, and includes an additional idle backoff slot at the end of a transmission or collision. According to this definition, a station whose backoff counter, during a given slot, is equal to b, will transmit in the next b-th slot.

Let us define with the term "Backoff Stage" the number of retransmissions suffered by an Head-Of-Line (HOL) packet (note that this definition differs from what adopted in [1]). The backoff stage ranges from a minimum value 0 (first transmission), to a maximum value R representing the maximum number of retries after which a frame is dropped from the transmission buffer. The case of infinite retries is modeled by setting $R = \infty$. Owing to the saturation assumption, a station in backoff stage 0, i.e., willing to transmit a new packet, will draw a random backoff value from a random variable b_0, uniformly distributed in the range $(0, CW_0)$. If the transmitted packet collides, the next backoff value extracted in stage 1, will be drawn from a random variable b_1, in a possibly different range $(0, CW_1)$ and so forth[1].

III. Alternative Derivation

Let us denote with (TX) the event that a station is being transmitting a frame into a time slot, and denote with $(s = i)$ the event that the station is found in backoff stage $i \in (0, \cdots, R)$. We are ultimately interested in the unconditional probability $\tau = P(TX)$ that the station transmits in a randomly chosen slot. Thanks to Bayes' theorem,

$$P(TX) \frac{P(s = i|TX)}{P(TX|s = i)} = P(s = i) \qquad i \in (0, \cdots, R). \tag{1}$$

Since this equality holds for all is in $(0, \cdots, R)$, it also holds for the summation:

$$P(TX) \sum_{i=0}^{R} \frac{P(s = i|TX)}{P(TX|s = i)} = \sum_{i=0}^{R} P(s = i) = 1. \tag{2}$$

We can thus express τ as:

$$\tau = P(TX) = \frac{1}{\sum_{i=0}^{R} \frac{P(s=i|TX)}{P(TX|s=i)}}. \tag{3}$$

[1]The specific exponential backoff rules adopted in DCF are modeled by setting $CW_i = min(2^i(CW_{min} + 1) - 1, CW_{max})$ for $i = 1 \cdots R$, and, for reasons that will be clarified in the following section IV, $CW_0 = CW_{min} - 1$ (instead of the more intuitive value $CW_0 = CW_{min}$).

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The conditional probability $P(s=i|TX)$ represents the probability that a station being transmitting is found in stage i. This probability is the steady-state distribution of a discrete-time Markov chain $s(k)$, describing the evolution of the backoff stage during the station's transmission instants k, and whose non-null one-step transition probabilities are:

$$\begin{cases} P(s(k+1)=i|s(k)=i-1)=p & i=1,\cdots,R \\ P(s(k+1)=0|s(k)=i)=1-p & i=0,\cdots,R-1 \\ P(s(k+1)=0|s(k)=R)=1 & i=R \end{cases} \quad (4)$$

where p, referred to as conditional collision probability, is the probability that a packet transmitted shall collide. Following [1], p is assumed to be a constant value, independent of the number of retransmissions occurred. It readily follows that $P(s=i|TX)$ is a (truncated, in the case of finite value R) geometric distribution, i.e.:

$$P(s=i|TX)=\frac{(1-p)p^i}{1-p^{R+1}} \qquad i\in(0,\cdots,R). \quad (5)$$

we remark that the generalization to more complex backoff processes with memory, i.e., whose backoff stage evolution is regulated by a Markov chain (e.g. the slow CW decrease proposed in [3] is one of such cases) is immediate, by defining an alternative Markov chain for process $s(k)$ and substituting its steady-state distribution in (5).

Let us now focus on the conditional transmission probability $P(TX|s=i)$, i.e., the probability that a station transmits while being in backoff stage i. We can envision the transmission process as the recurrence of consecutive transmission cycles, composed by transmission events separated by backoff times. Since we are conditioning on a given backoff stage i, we have to consider a sub-set of cycles, corresponding to backoff times and transmissions originated while in stage i. Assuming independence among transmission cycles, from renewal theory[2] we conclude that the probability $P(TX|s=i)$ can be computed by dividing the average number of slots spent for transmissions in a transmission cycle (owing to the time scale adopted, exactly 1 slot), with the average number of slots spent by the station during the whole cycle. Since, according to the definition given in section II, a model slot corresponds to a backoff counter decrement, it readily follows that:

$$P(TX|\,s=i)=\frac{1}{1+E[b_i]} \qquad i\in(0,\cdots,R). \quad (6)$$

where $E[b_i]$ is the average value of the backoff counter extracted by a station entering stage i. $E[b_i]$ results equal to $CW_i/2$ in the assumption of uniform distribution in the range $(0,CW_i)$. Substituting (5) and (6) into (3):

$$\tau=\frac{1}{\sum_{i=0}^{R}\frac{1-p}{1-p^{R+1}}p^i\left(1+E[b_i]\right)}=\frac{1}{1+\frac{1-p}{1-p^{R+1}}\sum_{i=0}^{R}p^iE[b_i]} \quad (7)$$

We remark that, in the above derivation, the transmission probability τ depends only on the sequence of mean backoff values $E[b_i]$, and not on the specific probability distributions $P(b_i=k)$ from which the backoff counters are extracted. This "insensitivity" property implies that different backoff distributions with the same average values will give identical throughput performance. Moreover, τ depends on the conditional collision probability p, which can be expressed as the probability that, in a time slot, at least one of the $n-1$ remaining stations transmits. At steady state, each remaining station transmits a packet with probability τ. This yields:

$$p=1-(1-\tau)^{n-1} \quad (8)$$

Equations (7) and (8) represent a non linear system in the two unknowns τ and p, which can be solved using numerical techniques.

[2]Specifically, this computation can be interpreted as an application of the Long-Run Renewal rate theorem (see, e.g. William Feller, An introduction to probability theory and its Applications, Vol. II, Wiley, Cap. XI - pp. 368-380).

IV. THROUGHPUT

Once the value τ is known, the throughput S can be computed as [2]:

$$S=\frac{P_sP_{tr}E[P]'}{(1-P_{tr})\sigma+P_{tr}P_sT'_s+P_{tr}(1-P_s)T'_c} \quad (9)$$

where $P_{tr}=1-(1-\tau)^n$ is the probability that there is at least one transmission in the considered slot time, $P_s=n\tau(1-\tau)^{n-1}/P_{tr}$ is the probability that a transmission occurring on the channel is successful, $E[P]'$ is the average amount of payload bits transmitted in a transmission slot, σ is the DCF backoff slot size, and T'_s and T'_c are the average successful transmission slot time and the average collision slot time, respectively. We recall that the denominator in (9) represents the average slot size $E[slot]$.

The values $E[P]'$, T'_s and T'_c slightly differ with respect to the corresponding ones defined in [2], in order to more accurately model the backoff freezing operation specified in the 802.11 DCF standard. $E[P]'$ and T'_s are derived as follows. Neglecting capture effects, a successful transmission implies that all the other "listening" stations have a backoff counter greater or equal than 1 at the beginning of the transmission period (otherwise they would have been involved in the considered transmission period and a collision would have occurred). According to the DCF specifications, after a DIFS time, a listening station will decrement its backoff counter only after a further idle backoff slot σ has elapsed. Hence, only the successfully transmitting station may access the first slot after the DIFS. This occurs when it extracts a new backoff counter value equal to zero, i.e. with probability $B_0=1/(CW_{min}+1)$. Moreover, such an eventual transmission will be collision-free. Consistently with the new definition of model slot given before, we may conclude that a successful transmission slot may contain more than a single packet. Being $E[P]$ the average packet payload size, the amount of information transmitted in a successful access is thus:

$$E[P]'=E[P]+\sum_{i=1}^{\infty}B_0^kE[P]=\frac{E[P]}{1-B_0}, \quad (10)$$

and the duration of a successful transmission slot is updated, with respect to the T_s value defined in [2], as:

$$T'_s=T_s+\sum_{i=1}^{\infty}B_0^kT_s+\sigma=\frac{T_s}{1-B_0}+\sigma, \quad (11)$$

where an extra empty backoff slot σ is included in T'_s, to allow listening stations to decrement their backoff counter (thus ending the model slot as defined in section II). This also implies that a successfully transmitting station will enter a new model slot with the initially drawn backoff counter (which is greater than 0 because of our slot definition) decremented of one unit, and thus in the range $(0, CW_{min} - 1)$.

Similar considerations can be drawn for what concerns T_c', which is updated as $T_c' = T_c + \sigma$, where T_c can be computed either as in [2], or by considering an EIFS instead of a DIFS after the end of the collision (this depends on the assumptions made on the physical layer operation[3]). In fact, neither the listening stations, for the reasons discussed above, nor the contending stations, due to the very specific setting of the ACK_Timeout parameter in the DCF specification[4], may access the first backoff slot σ at the end of the considered DIFS or EIFS interframe space.

V. Delay Derivation via Little's Result

Let D be the average access delay, defined as the time elapsing between the instant of time the packet is put into service - i.e., it becomes head-of-line (HOL) - and the instant of time the packet terminates a successful deliver. D can be computed via Little's Result as:

$$D = \frac{E[N]}{S/E[P]} \tag{12}$$

where the numerator $E[N]$ represents the average number of competing stations which will successfully deliver their HOL packet, and the denominator represents the packet delivery rate (i.e., the throughput measured in packets/seconds). In the case of unbounded retries ($R = \infty$), all the competing stations will ultimately deliver their HOL packet, and thus it is trivial to conclude that $E[N] = n$. Conversely, it is not obvious to determine the proper value $E[N]$ in the case of finite retry limit, since some of the n packets competing during a randomly chosen slot-time will be ultimately dropped. In such a case,

$$E[N] = n[1 - P\,(pck\ drop)] \tag{13}$$

where $P\,(pck\ drop)$ represents the probability that a competing station, randomly chosen in a generic slot-time, will ultimately lose its HOL packet due to retry limit exhaustion. Since this probability depends on the number of already suffered transmissions, conditioning on the probability $P(s = i)$ that in the randomly chosen slot the station is found in backoff stage i, we obtain:

$$P\,(pck\ drop) = \sum_{i=0}^{R} P\,(pck\ drop|s = i)\, P\,(s = i) \tag{14}$$

A packet in backoff stage i will be dropped if it first reaches stage R (i.e., it collides for $R - i$ times) and then it also collides during the last transmission attempt. Hence, $P\,(pck\ drop|s = i) = p^{R+1-i}$. The probability $P(s = i)$, given in (1), can be expressed in terms of the known values $p, \tau, E[b_i]$, and R through (5) and (6), yielding:

$$P\,(s = i) = \tau \cdot \frac{(1-p)p^i}{1 - p^{R+1}} \cdot (1 + E[b_i]) \tag{15}$$

Hence, after algebraic simplifications,

$$P\,(pck\ drop) = \tau(1-p)\frac{p^{R+1}}{1-p^{R+1}} \sum_{i=0}^{R}(1 + E[b_i]) \tag{16}$$

and, finally,

$$D = \frac{n}{S/E[P]} - E[slot](1 - B_0)\frac{p^{R+1}}{1-p^{R+1}} \sum_{i=0}^{R}(1 + E[b_i]) \tag{17}$$

where, in the simplifications, we have made use of the fact that:

$$\tau(1-p) = \frac{S \cdot E[slot]}{nE[P]'} = \frac{S(1 - B_0)}{nE[P]} E[slot] \tag{18}$$

which is immediate from (9) and (10). The average delay expression (17) is consistent with the one found in [10]. It takes into account only the packets successfully delivered at the destination, while packets dropped because of frame retry limit do not contribute to the delay computation

[3]The usage of an EIFS or a DIFS depends, respectively, on whether a listening station succeeds or not in bit-synchronizing during the transmission of the colliding physical layer preambles. For numerical comparison purposes, we alert the reader that some common simulation platforms (e.g. OPNET and NS-2) rely on the EIFS assumption.

[4]According to the formal description of the MAC operation, appendix of the standard, Ack_Timeout = CTS_Timeout = aSifs + Duration(Ack) + PLCPHeader + PLCPPreamble + aSlotTime.

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