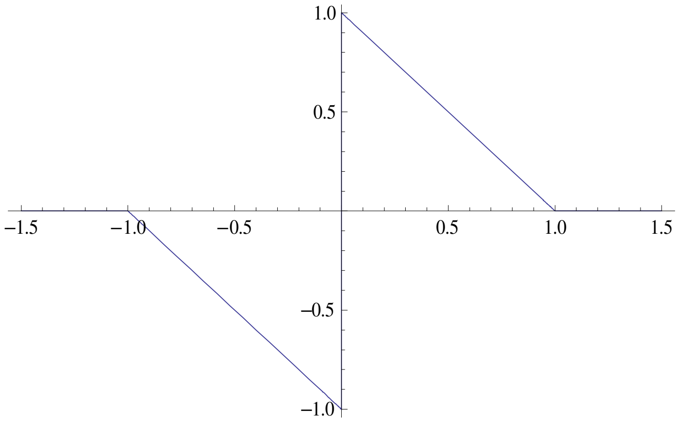
Fondamenti di comunicazioni elettriche (Ing. Elettronica - A.A.2011-2012)

Esercizio 1 [**]

$$g_1(t) = A(1-t)\,\mathrm{rect}(t-1/2) - A(1+t)\,\mathrm{rect}(t+1/2)$$

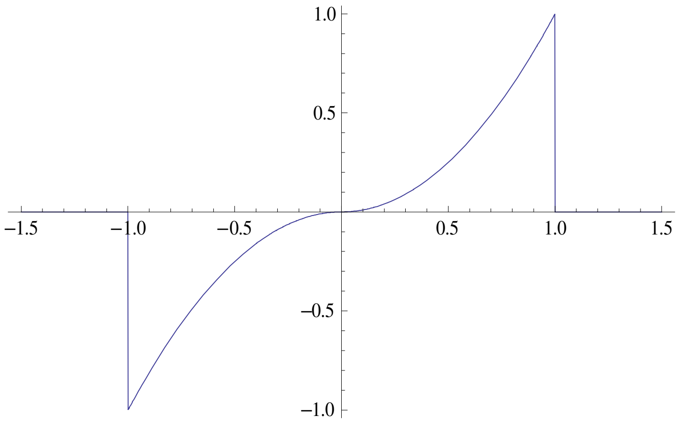


Soluzione

$$g_1'(t) = -A\,\mathrm{rect}(t/2) + 2A\delta(t)$$
$$j2\pi f G_1(f) = -2A\,\mathrm{sinc}(2f) + 2A$$
$$G_1(f) = A\frac{1-\mathrm{sinc}(2f)}{j\pi f}$$
$$G_1(0) = \int_{-\infty}^{\infty} g_1(t)dt = 0$$

Esercizio 2 [**]

$$g_2(t) = t^2\,\mathrm{sign}(t)\,\mathrm{rect}(t/2)$$

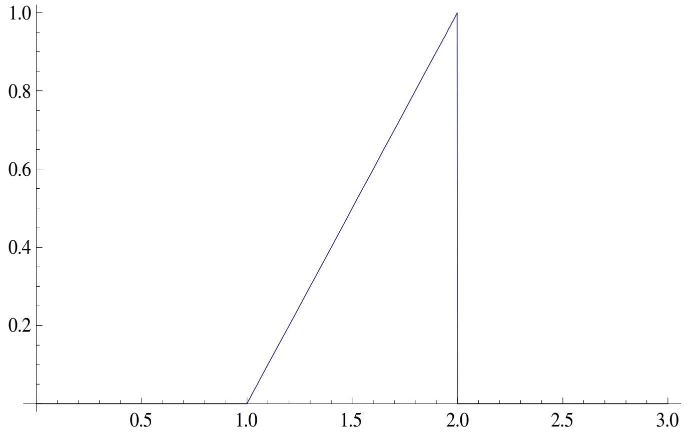


Soluzione

$$g_2'(t) = 2t\,\mathrm{rect}\left(t-1/2\right) - 2t\,\mathrm{rect}(t+1/2) - \delta(t-1) - \delta(t+1)$$
$$g_2''(t) = 2\,\mathrm{rect}\left(t-1/2\right) - 2\delta(t-1) - 2\,\mathrm{rect}(t+1/2) + 2\delta(t+1) - \delta'(t-1) - \delta'(t+1)$$
$$-4\pi^2 f^2 G_2(f) = 2\,\mathrm{sinc}(f)e^{-j\pi f} - 2e^{-j2\pi f} - 2\,\mathrm{sinc}(f)e^{j\pi f} + 2e^{j2\pi f} - j2\pi f e^{-j2\pi f} - j2\pi f e^{j2\pi f}$$
$$G_2(f) = \frac{-\mathrm{sinc}^2(f) + 2\,\mathrm{sinc}(2f) - \cos(2\pi f)}{j\pi f}$$
$$G_2(0) = \int_{-\infty}^{\infty} g_2(t)dt = 0$$

Esercizio 3 [**]

$$g_3(t) = (t-1)\,\mathrm{rect}(t-3/2)$$

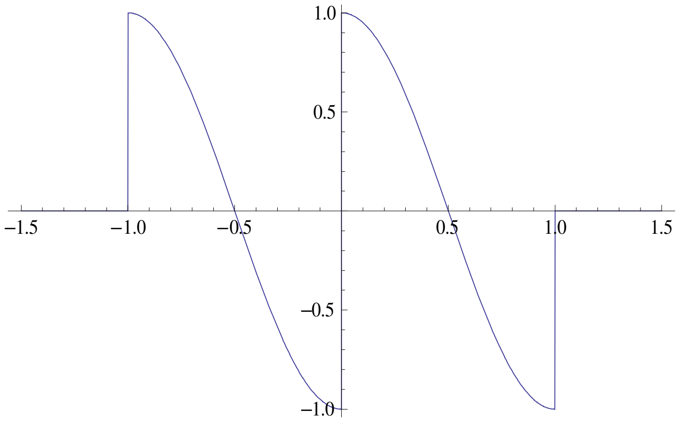


Soluzione

$$g_3'(t) = \mathrm{rect}(t-3/2) - \delta(t-2)$$
$$j2\pi f G_3(f) = \mathrm{sinc}(f)e^{-j3\pi f} - e^{-j4\pi f}$$
$$G_3(f) = \frac{\mathrm{sinc}(f)e^{-j3\pi f} - e^{-j4\pi f}}{j2\pi f}$$
$$G_3(0) = \int_{-\infty}^{\infty} g_3(t)dt = 1/2$$

Esercizio 4 [**]

$$g_4(t) = \cos(\pi t)\left[\mathrm{rect}(t-1/2) - \mathrm{rect}(t+1/2)\right]$$

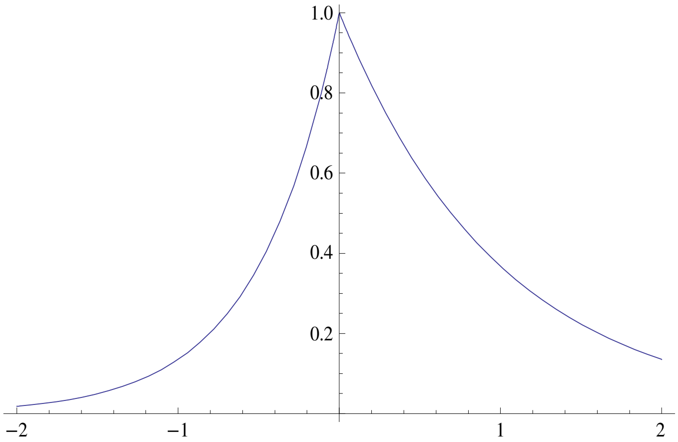


Soluzione

$$G_4(f) = \left[\frac{1}{2}\delta(f-1/2) + \frac{1}{2}\delta(f+1/2)\right] * \left[\mathrm{sinc}(f)e^{-j\pi f} - \mathrm{sinc}(f)e^{j\pi f}\right]$$
$$= \left[\frac{1}{2}\delta(f-1/2) + \frac{1}{2}\delta(f+1/2)\right] * \left[-j2\pi f\,\mathrm{sinc}^2(f)\right]$$
$$= -j\pi\left[(f-1/2)\,\mathrm{sinc}^2(f-1/2) + (f+1/2)\,\mathrm{sinc}^2(f+1/2)\right]$$

Esercizio 5 [*]

$$g_5(t) = e^{-t}\,\mathrm{u}(t) + e^{2t}\,\mathrm{u}(-t)$$

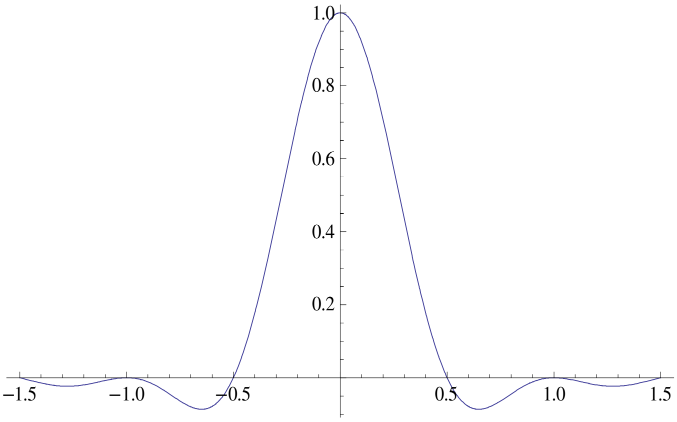


Soluzione

$$G_5(f) = \frac{1}{1+j2\pi f} + \frac{1}{\left|-2\right|}\frac{1}{1+j2\pi\dfrac{f}{-2}} = \frac{1}{1+j2\pi f} + \frac{1}{2-j2\pi f}$$

Esercizio 6 [***]

$$g_6(t) = \mathrm{sinc}(t)\,\mathrm{sinc}(2t)$$

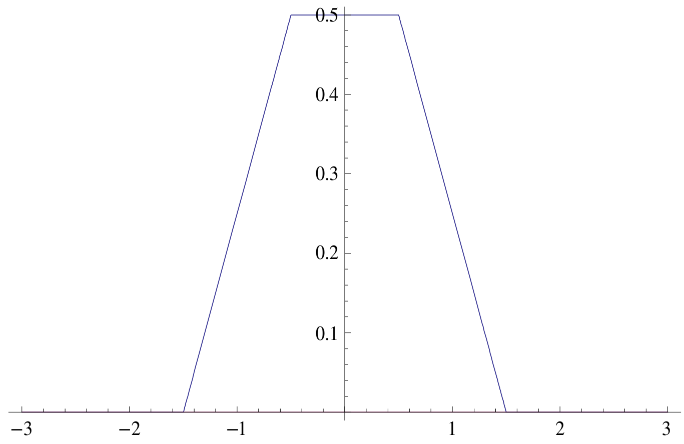


Soluzione

$$G_6(f) = \mathrm{rect}(f) * \frac{1}{2}\mathrm{rect}(f/2)$$

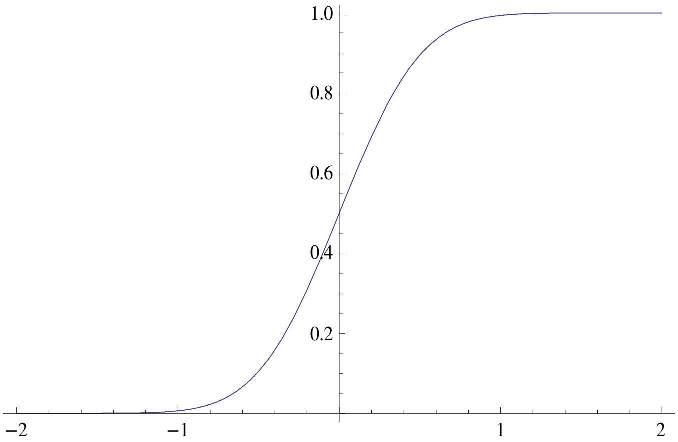
$$\begin{aligned} G_6'(f) &= \mathrm{rect}(f) * \frac{1}{2}(\delta(f+1) - \delta(f-1)) \\ &= \frac{1}{2}\mathrm{rect}(f+1) - \frac{1}{2}\mathrm{rect}(f-1) \end{aligned}$$

$$\begin{aligned} G_6(f) = \int_{-\infty}^{f} G_6'(\varphi)d\varphi &= \begin{cases} 0 & f < -3/2 \\ (f+3/2)/2 & f \in (-3/2,-1/2) \\ 1/2 & f \in (-1/2,1/2) \\ -(f-3/2)/2 & f \in (1/2,3/2) \\ 0 & f > 3/2 \end{cases} \\ &= \frac{1}{2}(f+3/2)\,\mathrm{rect}(f+1) + \frac{1}{2}\mathrm{rect}(f) + \frac{1}{2}(3/2-f)\,\mathrm{rect}(f-1) \end{aligned}$$


0.5
0.4
0.3
0.2
0.1
−3
−2
−1
1
2
3

Esercizio 7 [*]

$$g_7(t) = \int_{-\infty}^{t} e^{-\pi t^2} dt$$

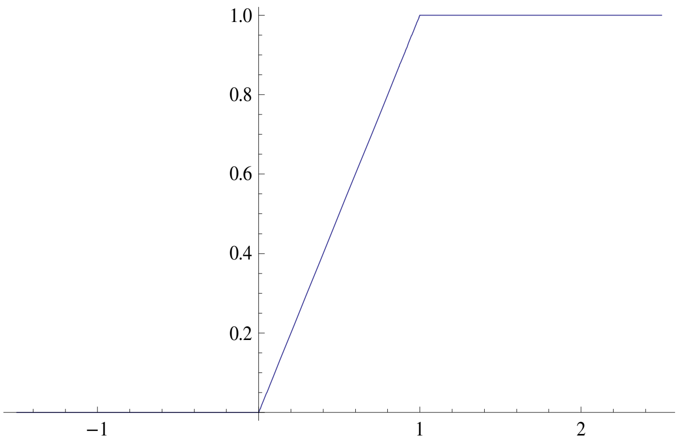


Soluzione

$$g_7'(t) = e^{-\pi t^2}$$
$$g_7(t) = e^{-\pi t^2} * \mathrm{u}(t)$$
$$G_7(f) = e^{-\pi f^2}\left(\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right) = \frac{1}{j2\pi f} e^{-\pi f^2} + \frac{1}{2}\delta(f)$$

Esercizio 8 [*]

$$g_8(t) = \mathrm{u}(t-1) + t\,\mathrm{rect}(t - 1/2)$$

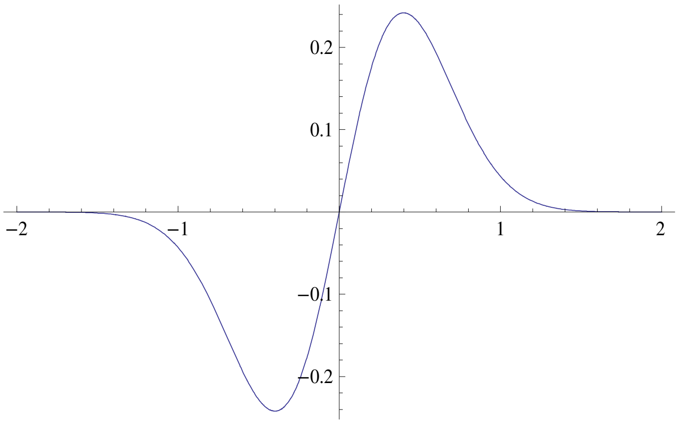


Soluzione

$$g_8'(t) = \mathrm{rect}(t - 1/2)$$
$$g_8(t) = \mathrm{rect}(t - 1/2) * \mathrm{u}(t)$$
$$G_8(f) = \mathrm{sinc}(f) e^{-j\pi f}\left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right] = \frac{\mathrm{sinc}(f) e^{-j\pi f}}{j2\pi f} + \frac{1}{2}\delta(f)$$

Esercizio 9 [**]

$$g_9(t) = te^{-\pi t^2}$$



Soluzione

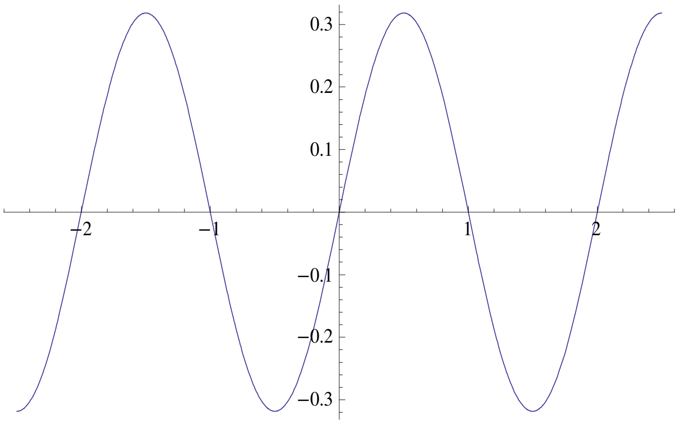
$$-j2\pi g_9(t) = -j2\pi t e^{-\pi t^2}$$

$$-j2\pi G_9(f) = \frac{d}{df} e^{-\pi f^2} = -2\pi f e^{-\pi f^2}$$

$$G_9(f) = -jfe^{-\pi f^2}$$

Esercizio 10 [**]

$$g_{10}(t) = t\,\mathrm{sinc}(t)$$



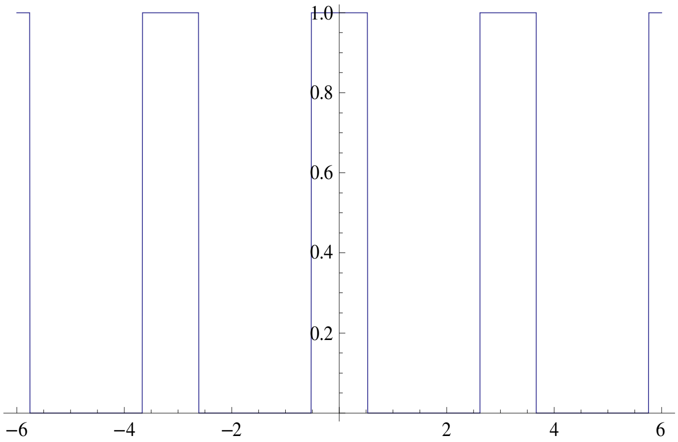
Soluzione

$$g_{10}(t) = \frac{1}{\pi}\sin(\pi t) = \frac{1}{j2\pi}(e^{j\pi t} - e^{-j\pi t})$$

$$G_{10}(f) = \frac{1}{j2\pi}\delta(f - 1/2) - \frac{1}{j2\pi}\delta(f + 1/2)$$

Esercizio 11 [****]

$$g_{11}(t) = \mathrm{rect}(\sin(t))$$



Soluzione

$$\left|\sin(t)\right| < 1/2 \Leftrightarrow t \in (-\pi/6 + k\pi, \pi/6 + k\pi)$$

$$g_{11}(t) = \sum_k \mathrm{rect}(\frac{t - k\pi}{\pi/3})$$

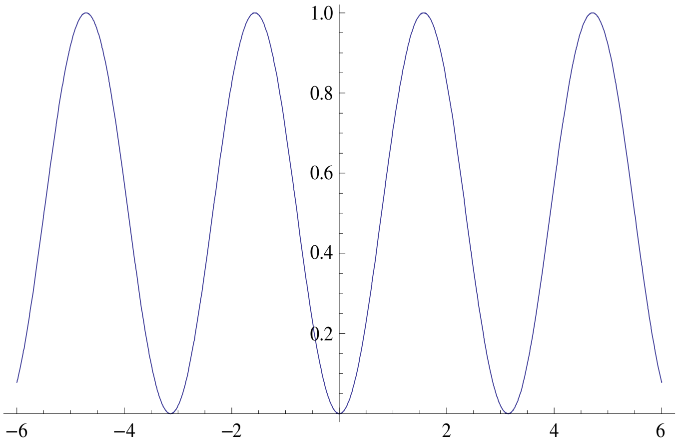
$$G_{11}(f) = \frac{\pi}{3}\mathrm{sinc}(f\pi/3)\sum_k e^{-j2\pi fk\pi} =$$

$$= \frac{\pi}{3}\mathrm{sinc}(f\pi/3)\sum_k \frac{1}{\pi}\delta(f - k/\pi) =$$

$$= \sum_k \frac{1}{3}\mathrm{sinc}(k/3)\delta(f - k/\pi)$$

Esercizio 12 [*]

$$g_{12}(t) = \sin^2(t)$$



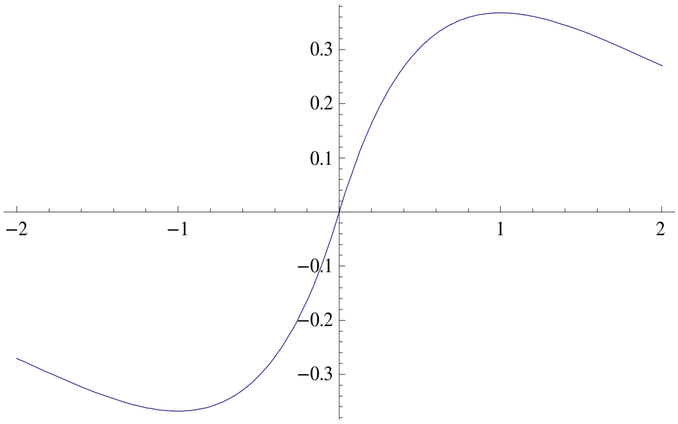
Soluzione

$$g_{12}(t) = \frac{1}{2} - \frac{1}{2}\cos(2t)$$

$$G_{12}(f) = \frac{1}{2}\delta(f) - \frac{1}{4}\delta(f - 1/\pi) - \frac{1}{4}\delta(f + 1/\pi)$$

Esercizio 13 [**]

$$g_{13}(t) = te^{-|t|}$$



Soluzione

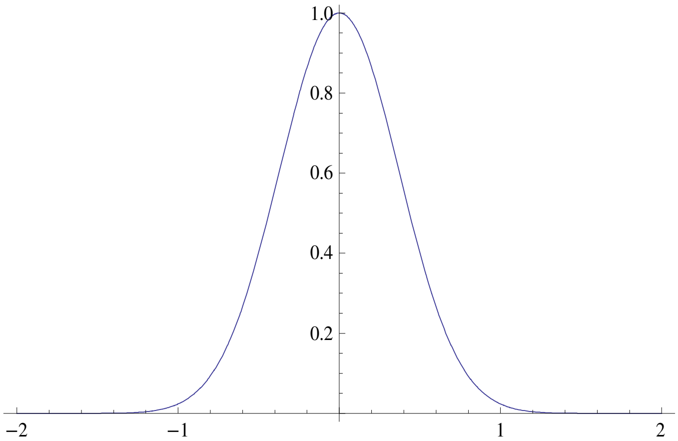
$$e^{-|t|} = e^{-t}\,\mathrm{u}(t) + e^{t}\,\mathrm{u}(-t)$$

$$\mathfrak{F}\left[e^{-|t|}\right] = \frac{1}{1+j2\pi f} + \frac{1}{1-j2\pi f} = \frac{2}{1+4\pi^2 f^2}$$

$$G_{13}(f) = \frac{1}{-j2\pi}\frac{d}{df}\frac{2}{1+4\pi^2 f^2} = \frac{8\pi f}{j(1+4\pi^2 f^2)^2}$$

Esercizio 14 [**]

$$g_{14}(t) = e^{-\pi t^2}\cos t$$



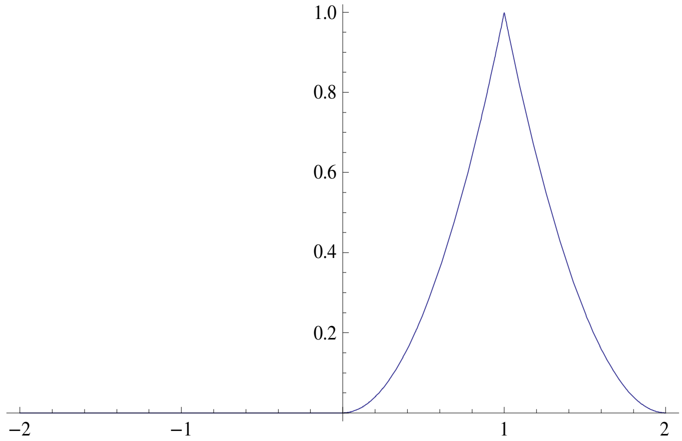
Soluzione

$$G_{14}(f) = e^{-\pi f^2} * \left[\frac{1}{2}\delta(f-\frac{1}{2\pi}) + \frac{1}{2}\delta(f+\frac{1}{2\pi})\right]$$

$$= \frac{1}{2}e^{-\pi(f-\frac{1}{2\pi})^2} + \frac{1}{2}e^{-\pi(f+\frac{1}{2\pi})^2}$$

Esercizio 15 [**]

$$g_{15}(t) = At^2 \,\mathrm{rect}(t - 1/2) + A(t-2)^2 \,\mathrm{rect}(t - 3/2)$$

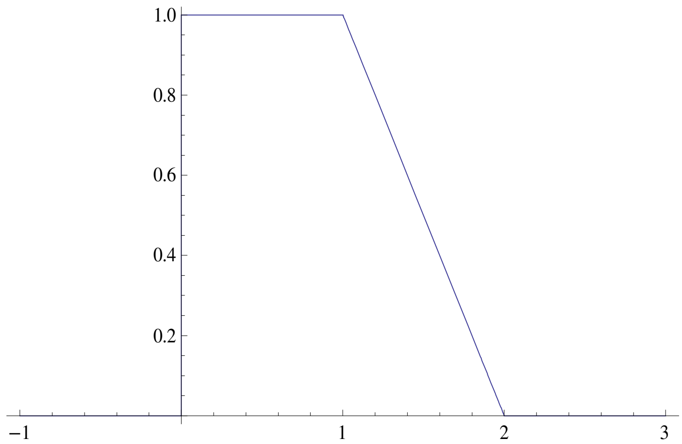


Soluzione

$$g'_{15}(t) = 2At \,\mathrm{rect}(t - 1/2) + 2A(t-2) \,\mathrm{rect}(t - 3/2)$$
$$g''_{15}(t) = 2A \,\mathrm{rect}((t-1)/2) - 4A\delta(t-1)$$
$$-4\pi^2 f^2 G_{15}(f) = 4A \,\mathrm{sinc}(2f) e^{-j2\pi f} - 4A e^{-j2\pi f}$$
$$G_{15}(f) = A e^{-j2\pi f} \frac{1 - \mathrm{sinc}(2f)}{\pi^2 f^2}$$
$$G_{15}(0) = 2A/3$$

Esercizio 16 [**]

$$g_{16}(t) = A \,\mathrm{rect}(t - 1/2) + A(2-t) \,\mathrm{rect}(t - 3/2)$$

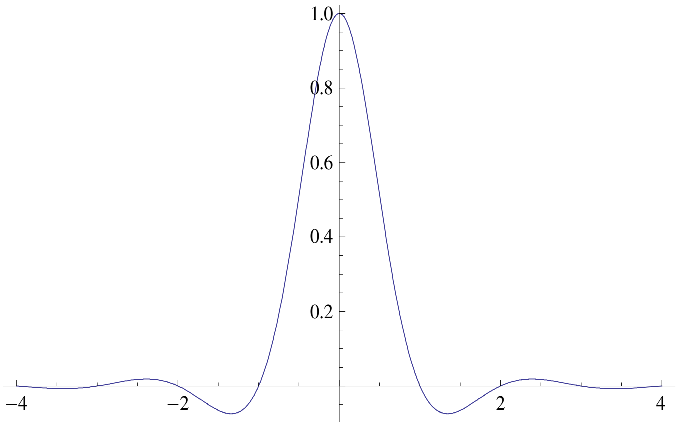


Soluzione

$$g'_{16}(t) = A\delta(t) - A \,\mathrm{rect}(t - 3/2)$$
$$j2\pi f G_{16}(f) = A - A \,\mathrm{sinc}(f) e^{-j3\pi f}$$
$$G_{16}(f) = A \frac{1 - \mathrm{sinc}(f) e^{-j3\pi f}}{j2\pi f}$$
$$G_{16}(0) = 3A/2$$

Esercizio 17 [*****]

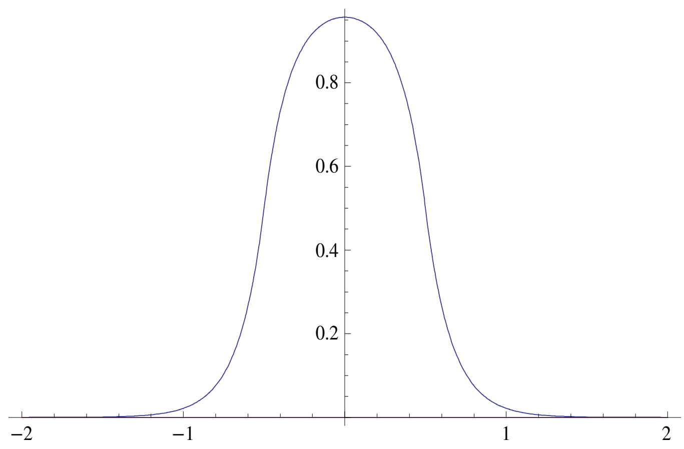
$$g_{17}(t) = \frac{\text{sinc}(t)}{1+t^2}$$



Soluzione

$$1+t^2 = 0 \Rightarrow t = \pm j$$
$$\frac{1}{1+t^2} = \frac{A}{1+jt} + \frac{B}{1-jt} = \frac{A+B-jAt+jBt}{1+t^2}$$
$$A+B=1$$
$$A-B=0$$
$$\frac{1}{1+t^2} = \frac{1}{2}\frac{1}{1+jt} + \frac{1}{2}\frac{1}{1-jt}$$
$$\mathfrak{F}[e^{-t}\,\mathrm{u}(t)] = \frac{1}{1+j2\pi f} \Rightarrow \mathfrak{F}[\frac{1}{1+j2\pi t}] = e^{f}\,\mathrm{u}(-f)$$
$$\mathfrak{F}[\frac{1}{1+jt}] = 2\pi e^{2\pi f}\,\mathrm{u}(-f)$$
$$\mathfrak{F}[\frac{1}{1-jt}] = 2\pi e^{-2\pi f}\,\mathrm{u}(f)$$
$$\mathfrak{F}\left[\frac{1}{1+t^2}\right] = \mathfrak{F}\left[\frac{1}{2}\frac{1}{1+jt} + \frac{1}{2}\frac{1}{1-jt}\right] =$$
$$= \left[\pi e^{2\pi f}\,\mathrm{u}(-f) + \pi e^{-2\pi f}\,\mathrm{u}(f)\right] = \pi e^{-2\pi|f|}$$

$$g_{17}(t) = \text{sinc}(t)\frac{1}{1+t^2}$$
$$G_{17}(f) = \text{rect}(f) * \pi e^{-2\pi|f|}$$
$$\text{rect}(f) = \mathrm{u}(f+1/2) - \mathrm{u}(f-1/2)$$
$$\pi e^{-2\pi|f|} * \mathrm{u}(f) = \int_{-\infty}^{f} \pi e^{-2\pi|\varphi|} d\varphi = \frac{1}{2}e^{2\pi f}\,\mathrm{u}(-f) + (1-\frac{1}{2}e^{-2\pi f})\,\mathrm{u}(f)$$
$$G_{17}(f) = \begin{cases} e^{2\pi f}\sinh(\pi) & f < -1/2 \\ 1 - e^{-\pi}\cosh(2\pi f) & f \in (-1/2, 1/2) \\ e^{-2\pi f}\sinh(\pi) & f > 1/2 \end{cases}$$



0.8
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2